Quo vadis: Hydrologic inverse analyses using high-performance computing and a D-Wave quantum annealer

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Unclassified: LA-UR-17-31098

Quantum Annealing (QA)

$$ f(q) = \sum_{i=1}^{n} a_i q_i + \sum_{i=1}^{n-1} b_{ij} q_i q_j $$

Returns q’s that make f(q) small.

Small 1D Inversion with QA

Finite difference equation: $$ 0 = \nabla \cdot (k \nabla h) $$

$$ 0 = k_1 (h_1 - h_2) + k_2 (h_3 - h_2) $$

Reformulate as a least squares problem

$$ 0 \approx [k_1 (h_1 - h_2) + k_2 (h_3 - h_2)]^2 $$

Fill in, say, $$ h_1 = 1, h_2 = \frac{1}{2}, h_3 = 0 $$

$$ 0 \approx \left( \frac{2k_1}{3} - k_2 \right)^2 $$

Discretize $$ k_i = 1 + q_i $$

$$ 0 \approx \left( \frac{2 + 2q_i}{3} - \frac{1 + q_2}{3} \right)^2 = \frac{8}{9} q_1 - \frac{1}{9} q_2 - \frac{4}{9} q_1 q_2 + \frac{1}{9} $$

$$ h_1 \cdot k_1 \cdot h_2 \cdot k_2 \cdot h_3 $$

$$ \frac{8}{9} - \frac{1}{9} - \frac{4}{9} \cdot \frac{8}{9} $$

$$ \frac{2}{9} \cdot \frac{8}{9} $$

$$ f(q_1, q_2) = \frac{8}{9} q_1 - \frac{1}{9} q_2 - \frac{4}{9} q_1 q_2 + \frac{1}{9} $$

$$ P(Q = q_1, Q = q_2) \propto \exp[-\beta f(q_1, q_2)] $$

$$ \beta \approx 16.6 $$

The identification problem as stated in the present work is solved as a linear or a quadratic programming problem. The solution in the latter case is much more complicated, whereas the solution of the linear programming problem is based on readily available computer programs.\(^1\)

References